STUDY GUIDE:

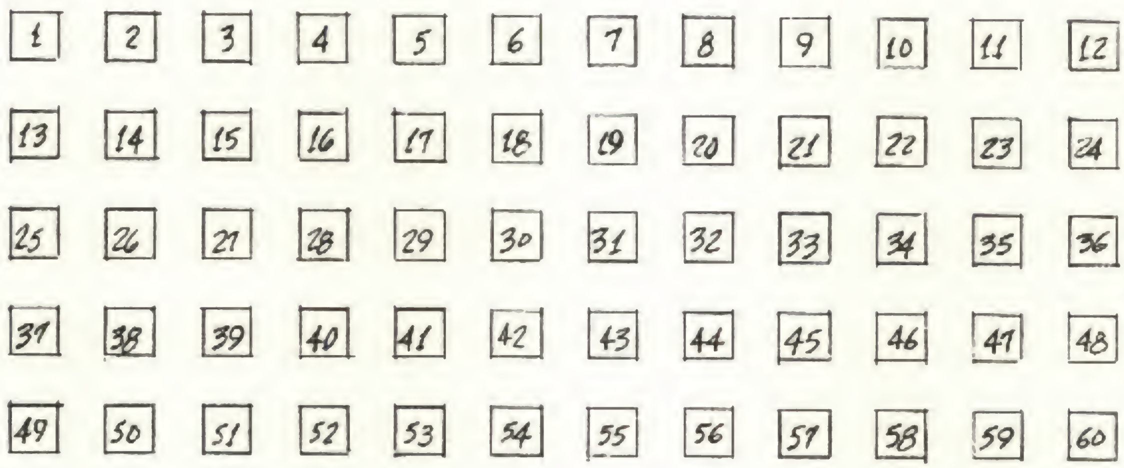
Module 4: Rational Numbers, Part I:

Much of mathematics hinges on the study of *rates*. To find a rate we divide one number by another. For example, if we take 6 apples and divide it by 3 doctors, we get 2 *apples per doctor*. "Apples per doctor" is a rate. More generally, a rate is always named by two nouns separated by the word, "per". Because rates are so important, we give them special treatment in mathematics. In particular, when we divide any whole number by any non-zero whole number, we call the quotient a *rational number*. "Rational" is derived from the same word as is "rate" and "ratio". This module is the first in a 5-module series dealing with rational numbers.

In this module we introduce rational numbers in terms of taking fractional parts of an amount. This concept requires only that we know how to multiply and divide whole numbers. For example if you wanted half of 300 pieces of candy you would divide 300 by 2. "Half" means that you are taking 1 piece out of each 2. If you wanted to take 1 piece out of each 3, you'd divide 300 by 3. The quotient would be called *one-third* of 300 and would be written as $\frac{1}{3}$ of 300. If you had wished to take 2 out of each 3 pieces we would first divide 300 by 3 and then multiply the quotient by 2. The answer would be called *two-thirds* of 300 and is written as $\frac{2}{3}$ of 300.

An expression such as $\frac{2}{3}$ is called a *common fraction*. The bottom number is called the *denominator*. It is the number we divide by. It is called the denominator because it determines the size (*denomination*) of each part. The top number is called the *numerator*. It is the number we multiply by. It counts (*enumerates*) the number of parts we've taken. For example to take $\frac{4}{5}$ of 60, we divide 60 by 5 to get 12; and we then multiply 12 by 4 to get 48. This tells us that at a rate of 4 out of each 5, we'd take 48 out of a group of 60.

From a more pictorial point of view, look at the diagram below. Each of the 60 items is represented by a small square. To indicate that we have divided by 5, we have arranged the squares in 5 rows, with 12 items each. To indicate that we then multiplied by 4, we have shaded 4 of the 5 rows of squares.



If we now look at the columns rather than at the rows, it is easy to see that we have taken 4 out of each 5 items (per column).

The rest of the module is concerned with comparing, adding, or subtracting two or more common fractions.

Step 1:

View Videotape Lecture #4.

Step 2:

Read Module 4 of the Text.

Step 3:

When you feel that you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

Check The Main Ideas

$\frac{4}{7}$ is called a common __________. We call 7	*fraction*
the __________ of the common fraction and 4 is	*denominator*
called the ____________. We read $\frac{4}{7}$ as 4 ________.	*numerator; sevenths*
To take $\frac{4}{7}$ of an amount means that we want to	
take ____ out of each ______. To see how much this	*4; 7*
is, we could first divide the amount by _____ and	*7*
then multiply by _____. For example, to find out	*4*
how much $\frac{4}{7}$ of 35 is, we first divide 35 by _____	*7*
and then multiply this by ____ to get 20. In other	*4*
words, at a rate of 4 out of each 7, we would take	
20 out of each _____. In the language of common	*35*
fractions we'd write this as $\frac{4}{7}$ = _____. In	$\frac{20}{35}$
getting from $\frac{4}{7}$ to $\frac{20}{35}$ we multiplied both numerator	
and denominator of $\frac{4}{7}$ by ______.	*5*
Whenever we _______ numerator and denominator	*multiply*
by the same non-zero number we get an equivalent	
common fraction. For example if we multiply	
numerator and denominator of $\frac{3}{5}$ by 7 we get the	
equivalent common fraction ______. That is,	$\frac{21}{35}$
a rate of 3 out of each 5 is the same as a rate of	
____ out of each 35. To compare the size of two	*21*
common fractions we look for equivalent common	
fractions that have a common (that is, the same)	
_____________.	*denominator*

For example suppose we wanted to compare $\frac{3}{4}$ and $\frac{7}{11}$; that is, 3 ________ and 7 __________.	*fourths; elevenths*
The fact that 3 is less than 7 doesn't help us here. In other words 3 quarters is _____ money than is 7 dimes. So to compare the size of two common fractions just by looking at their numerators requires that the fractions have the same __________.	*more* *denominator*
Since 4 X 11 = 44, a common denominator for $\frac{3}{4}$ and $\frac{7}{11}$ would be _____. So we can multiply numerator and denominator of $\frac{3}{4}$ by 11 to get _____;	*44* $\frac{33}{44}$
and we can multiply numerator and denominator of $\frac{7}{11}$ by ___ to get $\frac{28}{44}$. That is:	*4*
$\frac{3}{4} = 33$ ______________	*forty-fourths (44ths)*
and $\frac{7}{11} = 28$ ______________	*forty-fourths*
Since 33 - 28 = ____, we see that the difference	*5*
between $\frac{3}{4}$ and $\frac{7}{11}$ is 5 ____________; or in the language of subtraction:	*forty-fourths*
$\frac{3}{4} - \frac{7}{11} =$ ______________	$\frac{5}{44}$ $(\frac{33}{44} - \frac{28}{44})$
In a similar way, the fact that 33 + 28 = 61 means that $\frac{3}{4} + \frac{7}{11} = \frac{33}{44} +$ _____ or $\frac{61}{44}$.	$\frac{28}{44}$
We can always find a common multiple of two whole numbers simply by _________ the two numbers.	*multiplying*
For example if we want a common multiple of 30 and 36, we can form the product 30 X ____ or 1,080.	*36*
1,080 is the ______ of 30 and the ______ multiple of 36.	*36th; 30th*

Now the first seven multiples of 30 are

30, 60, 90, 120, 150, 180, and _______; while *210*

the first seven multiples of 36 are 36, 72, 108,

144, 180, 216, and _______. Comparing these two *252*

lists we see that the least non-zero common multiple

of 30 and 36 is _______. To get this result *180*

without having to list the multiles requires that

we write both 30 and 36 as products of _______ *prime*

numbers. 2 is a prime number because its only

whole number factors are 2 and 1. 4 is not a

prime number because in addition to being 4 X 1 or

1 X 4 it can also be factored as _______. In *2 X 2*

any event, we get:

30 = 2 X 3 X ____ *5*

and

36 = 2 X 2 X 3 X ______ *3*

So any multiple of 30 must contain as a factor

2 X 3 X 5. Comparing this with 2 X 2 X 3 X 3, we

see that we need one more ___ and one more 3 as *2*

factors. This gives us 2 X 3 X 5 X 2 X 3 or _____ *180*

So to find the sum of $\frac{7}{30}$ and $\frac{5}{36}$ we could use

______ as the least common denominator. Since 180 is *180*

the 6th multiple of 30 we multiply numerator and

denominator of $\frac{7}{30}$ by 6 to get _______. And since *$\frac{42}{180}$*

180 is the 5th multiple of 36, we multiply numerator

and denominator of $\frac{5}{36}$ by ___ to get $\frac{25}{180}$. *5*

The fact that 42 + 25 = 67 means that

$\frac{42}{180} + \frac{25}{180}$ = ____________. Hence: *$\frac{67}{180}$*

$\frac{7}{30} + \frac{5}{36}$ = __________ *$\frac{67}{180}$*

As a check:

$\frac{1}{30}$ of 180 = 180 ÷ 30 or _____. *6*

Hence $\frac{7}{30}$ of 180 = 7 X ____ or 42. *6*

$\frac{1}{36}$ of 180 = 180 ÷ ____ or 5. *36*

Hence $\frac{5}{36}$ of 180 = 5 X ___ or 25. *5*

Therefore $\frac{7}{30}$ (of 180) + $\frac{5}{36}$ (of 180)

is equal to 42 + __ or 67 of the 180. *25*

In adding (or subtracting) common fractions it is crucial that you use common _________. DO NOT add numerators and denominators. For example: *denominators*

$\frac{2}{3} + \frac{5}{6}$

is not the same as

$\frac{2+5}{3+6}$ or __________. *$\frac{7}{9}$*

Rather we first rewrite the common fractions using common ____________. We then keep the common denominator and ______ the numerators. *denominators* *add*

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Step 4:

Do the Mastery Review on the next page.

Mastery Review

ANSWERS:

1. How much is $\frac{1}{2}$ of 108? — 1. ______

2. How much is $\frac{1}{3}$ of 21? — 2. ______

3. How much is $\frac{2}{3}$ of 750? — 3. ______

4. In the common fraction $\frac{5}{9}$ which number is: — 4. (a) ______

   (a) the denominator? (b) the numerator? — (b) ______

5. How much is $\frac{2}{3}$ of 120? — 5. ______

6. How much is $\frac{10}{15}$ of 120? — 6. ______

7. Find a common fraction whose numerator is is 30 that is equivalent to the common fraction: $\frac{5}{6}$ — 7. ______

8. Find a common fraction whose denominator is 30 that is equivalent to the common fraction: $\frac{5}{6}$ — 8. ______

9. Find a common fraction whose denominator is 30 that is equivalent to the common fraction $\frac{4}{5}$. — 9. ______

10. Based on Problems 8 and 9, which common fraction names the greater ratio: $\frac{5}{6}$ or $\frac{4}{5}$? — 10. ______

11. Which common fraction names the greater ratio, and by how much-- $\frac{3}{8}$ or $\frac{2}{5}$? — 11. ______

12. How much is $\frac{4}{9} + \frac{1}{9}$? — 12. ______

13. How much is $\frac{4}{9} + \frac{1}{5}$? — 13. ______

14. Find the sum of $\frac{5}{24} + \frac{3}{32}$. — 14. ______

15. Write 15 as the product of two whole numbers, neither of which is 1. — 15. ______

16. Can 5 be written as the product of two whole numbers, neither of which is 1? — 16. ______

17. (a) Is 5 a prime number? — 17. (a) ______

    (b) Is 15 a prime number? — (b) ______

Mastery Review (cont)

18. Is 2,600 a prime number? 18. ____________

19. Is 297 a prime number? 19. ____________

20. Is 10,149 a prime number? 20. ____________

21. Is 211 a prime number? 21. ____________

22. Write 40 as a product of whole numbers in which each number is a prime number. 22. ____________

23. What number is named by 4^3? 23. ____________

24. What number is named by 3^4? 24. ____________

25. Write 360 as a product of prime numbers. 25. ____________

26. Write 504 as a product of prime numbers. 26. ____________

27. Reduce $\frac{360}{504}$ to lowest terms. 27. ____________

28. Find the least common multiple of 72 and 96. 28. ____________

29. Find the sum of $\frac{5}{72}$ and $\frac{7}{96}$, making sure that your answer is in lowest terms. 29. ____________

30. Write $\frac{7}{96} - \frac{5}{72}$ as a common fraction in lowest terms. 30. ____________

Answers to Mastery Review

1. 54 2. 7 3. 500 4. (a) 9 (b) 7 5. 80 6. 80

7. $\frac{30}{36}$ *8.* $\frac{25}{30}$ *9.* $\frac{24}{30}$ *10.* $\frac{5}{6}$ *11.* $\frac{2}{5}$ *by* $\frac{1}{40}$ *12.* $\frac{5}{9}$

13. $\frac{29}{45}$ *14.* $\frac{29}{96}$ *15. 5 X 3 (or 3 X 5) 16. no 17. (a) yes (b) no*

18. no 19. no 20. no 21. yes 22. 2 X 2 X 2 X 5 or $2^3 \times 5$

23. 64 24. 81 25. 2 X 2 X 2 X 3 X 3 X 5 or $2^3 \times 3^2 \times 5$

26. 2 X 2 X 2 X 3 X 3 X 7 or $2^3 \times 3^2 \times 7$ *27.* $\frac{5}{7}$ *28. 288*

29. $\frac{41}{288}$ *30.* $\frac{1}{288}$

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Step 5:

Do Self-Test 4, Form A on the next page.

Self-Test 4, Form A

ANSWERS:

1. Which of the following is not a common fraction: $\frac{0}{6}$, $\frac{3}{3}$, $\frac{7}{4}$, $\frac{8}{0}$, or $\frac{1}{5}$?

   1. ______________

2. Which of the following common fractions is equivalent to 5/9

   $\frac{9}{5}$, $\frac{5+1}{9+1}$, $\frac{5+5}{9+9}$, $\frac{5 \times 5}{9 \times 9}$, or $\frac{5 \times 9}{9 \times 5}$?

   2. ______________

3. List the following common fractions in the order of increasing size; that is, from the least to the greatest.

   $\frac{3}{5}$, $\frac{13}{21}$, $\frac{2}{3}$, $\frac{4}{7}$, and $\frac{19}{30}$

   3. ______________

4. (a) How much is $\frac{2}{5} + \frac{1}{3}$?

   (b) You spend 2/5 of your weekly take-home pay on rent and 1/3 on food. If your weekly take-home pay is $300, how much do you spend each week on food and rent?

   4. (a) ______________

   (b) ______________

5. List all the prime numbers between 50 and 60.

   5. ______________

6. Write 23,100 as a product of prime numbers.

   6. ______________

7. (a) Find the least common multiple of 12, 18, and 45.

   (b) Express (5/12 + 7/18) + 2/45 as a common fraction in lowest terms.

   (c) Express 5/12 + (7/18 + 2/45) as a common fraction in lowest terms.

   7. (a) ______________

   (b) ______________

   (c) ______________

8. (a) Express (4/7 - 1/3) - 2/11 as a common fraction in lowest terms.

   (b) Express 4/7 - (1/3 - 2/11) as a common fraction in lowest terms.

   8. (a) ______________

   (b) ______________

9. Reduce $\frac{46}{667}$ to lowest terms.

   9. ______________

10. You want to buy 750 items that usually cost 39¢ each. A special sale advertises: "Buy 3; pay for only 2!" How much will the 750 items cost you during the special sale?

    10. ______________

(ANSWERS ARE ON NEXT PAGE)

Answers for Self-Test 4, Form A

1. $\frac{8}{0}$

2. $\frac{5 + 5}{9 + 9}$

3. $\frac{4}{7}$, $\frac{3}{5}$, $\frac{13}{21}$, $\frac{19}{30}$, and $\frac{2}{3}$

4. (a) $\frac{11}{15}$ (b) \$220

5. 53 and 59

6. $2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 11$

7. (a) 180 (b) $\frac{17}{20}$ (c) $\frac{17}{20}$ ($\frac{153}{180}$ reduces to $\frac{17}{20}$)

8. (a) $\frac{13}{231}$ (b) $\frac{97}{231}$

9. $\frac{2}{29}$

10. \$195

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If you did each problem in Form A correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 6.

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Step 6:

Study the solutions to Self-Test 4 on the following pages, giving special emphasis to any problems you failed to answer correctly.

Solutions for Self-Test 4, Form A

1.

The only thing that keeps $\frac{m}{n}$ from being a common fraction (where m and n are whole numbers) is if $n = 0$. That is, the only restriction on a common fraction is that *the denominator can't be 0.* Thus on our given list, $\frac{8}{0}$ is the only entry that is not a common fraction.

See why? The denominator is what we divide the amount by and we can't divide by 0. For example $2 \div 0 =$ ___ means the same as $0 \times$ ___ $= 2$, but any multiple of 0 is 0; hence it is impossible that $0 \times$ ___ $= 2$ can be true.

Let's look at the other choices one at a time.

(a) $\frac{0}{6}$ tells us to divide by 6 and then multiply by 0. Since any multiple of 0 is 0, $\frac{0}{6}$ is equivalent to 0. More visually, if we divide an amount into 6 equal parts and then take none of the equal parts, we've taken none of the objects. In symbols, if n is any non-zero whole number then:

$$\frac{0}{n} = 0$$

(b) $\frac{3}{3}$ tells us to divide by 3 and then multiply by 3. For example to find $\frac{3}{3}$ of 15, we first divide 15 by 3 to get 5; and then we multiply 5 by 3 to get 15. In other words, if we divide an amount into 3 equal parts and then take all 3 parts, we have taken the whole amount. In a similar way if we divided an amount into, say, 30 equal parts and then took all 30 parts, we'd again take the whole amount. In symbols, if n is any non-zero whole number,

$$\frac{n}{n} = 1$$

Solutions for Self-Test 4, Form A (cont)

1. (cont)

(c) $\frac{7}{4}$ means that we divide the amount into 4 equal parts and then take one of these parts, seven times. That is, to take $\frac{7}{4}$ of an amount we divide the amount by 4 and then multiply the quotient by 7. For example:

$$\frac{7}{4} \text{ of } 20$$

means that we divide 20 by 4 to get 5 and we then multiply 5 by 7 to get 35. In other words:

$$\frac{7}{4} \text{ of } 20 = 35$$

(d) $\frac{8}{0}$ is not a common fraction because we can't divide by 0.

(e) To take $\frac{1}{5}$ of any amount means to divide the amount by 5 and multiply the quotient by 1. Multiplying by 1 doesn't change the answer. Hence to take $\frac{1}{5}$ of a number simply means to divide that number by 5. More generally if n is any non-zero whole number:

$$\frac{1}{n} \text{ of an amount} = \text{the amount} \div n$$

Sometimes it takes a while to visualize a common fraction whose numerator is greater than its denominator. In fact such common fractions are often referred to as improper *fractions. Yet they still name ratios. For example suppose you make an investment in which you are told that for each \$4 you invest you'll receive a return of \$7. So if you invest \$4 five times (\$20), you expect a \$7 return five times (\$35). So all that happens is that if the numerator is greater than the denominator, your share is greater than the original amount.*

2.

Two common fractions are equivalent if they name the same ratio. $\frac{5}{9}$ means that we are taking 5 out of each 9. At the rate of 5 out of each 9, we'd take:

5 out of 9
10 out of 18
15 out of 27
20 out of 36

Too see how many this is in a given amount, we divide the amount by 9 and then multiply the quotient by 5.

5, 10, 15, 20 are multiples of 5 while 9, 18, 27, and 36 are multiples of 9.

2. (cont)

In the language of common fractions, we must multiply numerator and denominator by the same non-zero number to get an equivalent common fraction. In this sense we have:

$$\frac{5}{9} = \frac{5 \times 2}{9 \times 2} = \frac{5 \times 3}{9 \times 3} = \frac{5 \times 4}{9 \times 4}$$

That is:

$$\frac{5}{9} = \frac{10}{18} = \frac{15}{27} = \frac{20}{36} = \ldots\ldots.$$

The only member of our list that fits this criterion is $\frac{5+5}{9+9}$. That is $5 + 5 = 5 \times 2$ and $9 + 9 = 9 \times 2$; hence $\frac{5+5}{9+9} = \frac{5 \times 2}{9 \times 2}$

This may take awhile to get used to. Notice that to keep the rate the same we added 5 to the numerator and 9 to the denominator. We did not add the same number to both numerator and denominator.

Let's look at the other candidates one at a time.

(a) $\frac{9}{5}$ comes from $\frac{5}{9}$ by interchanging the numerator and denominator. These are different ratios. For example:

When we interchange the numerator and denominator of a fraction the new fraction is called the reciprocal of the original fraction. In other words, 9/5 is the reciprocal of 5/9 and vice versa

$$\begin{aligned} \frac{5}{9} \text{ of } 45 &= (45 \div 9) \times 5 \\ &= 5 \times 5 \\ &= 25 \end{aligned}$$

$$\begin{aligned} \frac{9}{5} \text{ of } 45 &= (45 \div 5) \times 9 \\ &= 9 \times 9 \\ &= 81 \end{aligned}$$

As a quick check, 5/9 is less than the whole amount, while 9/5 is greater than the whole amount.

(b) To get equivalent fractions we must multiply (or divide) numerator and denominator by the same non-zero amount. If we add the same amount, we usually change the ratio. For example $\frac{5+1}{9+1} = \frac{6}{10}$. $\frac{6}{10}$ and $\frac{5}{9}$ are not the same ratio. By way of illustration:

To keep the ratio the same in 5/9, for every 5 we add to the numerator, we must add 9 to the denominator.

2. (cont)

$\frac{6}{10}$ of $90 = (90 \div 10) \times 6$
$= 9 \times 6$
$= 54$

$\frac{5}{9}$ of $90 = (90 \div 9) \times 5$
$= 10 \times 5$
$= 50$

In other words $\frac{5}{9}$ is a rate of 50 out of 90 while $\frac{5+1}{9+1}$ is a rate of 54 out of 90.

We picked 90 because it is a common multiple of 9 and 10 (that is $10 \times 9 = 90$).

Note that we could have reduced 6/10 to 3/5 but this wasn't crucial to our discussion.

(c) We've already discussed why $\frac{5+5}{9+9}$ is equivalent to $\frac{5}{9}$.

(d) To keep the same ratio we must multiply numerator and denominator by the *same* non-zero number. In $\frac{5 \times 5}{9 \times 9}$ we are multiplying the numerator by 5 and the denominator by 9. This changes the ratio. For example $\frac{5 \times 5}{9 \times 9} = \frac{25}{81}$ but $\frac{5}{9} = \frac{5 \times 9}{9 \times 9} = \frac{45}{81}$. Clearly, 45 out of every 81 is not the same rate as taking 25 out of every 81.

If you're trying to memorize notice how similar (c) and (d) look. That's why it's important to take the time to understand the various concepts.

(e) $\frac{5 \times 9}{9 \times 5} = \frac{45}{45}$, which is the same as taking the entire amount. Clearly this is not the same as taking only 5 out of each 9.

This is a fairly interesting result. Namely: $\frac{m \times n}{n \times m} = \frac{m \times n}{m \times n} = 1$

Note that the main value of problems 1 and 2 is to help you visualize what ratios mean. If you try to memorize rather than understand, many wrong answers can seem to be right!

3.

The only completely safe way to compare the size of common fractions is to make sure they are rewritten in an form that have the same denominator. *The key point is that if two common fractions have equal denominators then the greater numerator names the greater fraction.*

Again, think in terms of money 4 nickels are more coins than 3 dimes, but 3 dimes has the greater purchasing power. But if we know that each coin is of the same denomination then 4 of the coins will always have more purchasing power than 3 of the coins.

So we look at the denominators and try to find a common multiple. The denominators are 5, 21, 3, 7, and 30. One way of finding a common multiple is to multiply all of these numbers; that is: 5 X 21 X 3 X 7 X 30 or 66,150 is a multiple of 5, 21, 3, 7, and 30.

A more convenient multiple can be found by using prime factorization. Namely

21 = 3 X 7 and 30 = 2 X 3 X 5

$$\begin{array}{r} 2\,\overline{)\,30} \\ 3\,\overline{)\,15} \\ 5 \end{array}$$

So any multiple of 21 needs 3 and 7 as factors while any multiple of 30 needs 2, 3, and 5 as factors. Hence a common multiple of 30 and 21 needs 2, 3, 5, and 7 as factors. Since 2 X 3 X 5 X 7 = 210, 210 is the least common multiple of 5, 21, 3, 7, and 30. In particular:

210 = 3 X 70
210 = 5 X 42
210 = 7 X 30
210 = 21 X 10
210 = 30 X 7

See how important it is to know the arithmetic of whole numbers in order to be able to study the arithmetic of common fractions.

3. (cont)

We now convert each common fraction into an ɜquivalent one in which the denominator is 210. Ve get:

$$\frac{3}{5} = \frac{3 \text{ X } 42}{5 \text{ X } 42} = \frac{126}{210}$$

$$\frac{13}{21} = \frac{13 \text{ X } 10}{21 \text{ X } 10} = \frac{130}{210}$$

$$\frac{2}{3} = \frac{2 \text{ X } 70}{3 \text{ X } 70} = \frac{140}{210}$$

$$\frac{4}{7} = \frac{4 \text{ X } 30}{7 \text{ X } 30} = \frac{120}{210}$$

$$\frac{19}{30} = \frac{19 \text{ X } 7}{30 \text{ X } 7} = \frac{133}{210}$$

Do you appreciate the value of working with 210 as the common denominator rather than working with 66,150?

In each case we multiply the numerator by whatever we multiplied the denominator by to get 210.

Now that we have common denominators we can ɪse the numerators for indicating the size. .anging from the least numerator to the greatest ɪe have:

126 is always less than 130 as long as 126 and 130 modify the same nouns (such as 210ths)

$$\frac{120}{210}, \quad \frac{126}{210}, \quad \frac{130}{210}, \quad \frac{133}{210}, \quad \frac{140}{210}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\frac{4}{7}, \quad \frac{3}{5}, \quad \frac{13}{21}, \quad \frac{19}{30}, \quad \frac{2}{3}$$

$\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{13}{21}$, and $\frac{19}{30}$ is in the order of ncreasing numerators but not in the order of ncreasing size (ratio).

Note that in the original form the numerators aren't helpful in determining the size of the frations. For example 3 fifths is more parts than 2 thirds, but 2 thirds is the greater share (140 per 210 versus 126 per 210)

4.

(a) To add two common fractions we need common denominator. The least common ultiple of 5 and 3 is 5 X 3 or 15. So we et:

$$\frac{2}{5} = \frac{2 \text{ X } 3}{5 \text{ X } 3} = \frac{6}{15}$$

4. (a) (cont)

$$\frac{1}{3} = \frac{1 \times 5}{3 \times 5} = \frac{5}{15}$$

Hence:

$$\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15}$$

$$= \frac{11}{15}$$

5 + 6 = 11 hence:
5 fifteenths + 6 fifteenths = 11 fifteenths

Do not add numerators and denominators to get

$$\frac{2}{5} + \frac{1}{3} = \frac{3}{8}$$

3 out of 8 is a much lower rate than 11 out of 15.

(b) It is not our aim this early in the course to deal with applications, but every once in a while an application can add reality to the arithmetic. In this respect part (b) is meant to bring part (a) "to life".

If your take-home pay is \$300 and you spend $\frac{2}{5}$ of it on rent, then the dollar-value of the rent is

$$\frac{2}{5} \text{ of } \$300 = (\$300 \div 5) \times 2$$
$$= \$60 \times 2$$
$$= \$120$$

In other words, at a rate of \$2 out of each \$5 you'd spend \$120 out of each \$300.

So you pay \$120 for rent.

Your food cost is $\frac{1}{3}$ of your weekly take-home pay, so in this case we have:

$$\frac{1}{3} \text{ of } \$300 = \$300 \div 3$$
$$= \$100$$

So you pay \$100 for food and \$120 for rent. Therefore, for both food and rent you pay \$220.

The point is that we could use part (a) to get the answer more efficiently. Namely:

4 (b) (cont)

We know that $\frac{2}{5}$ of the take-home pay added to $\frac{1}{3}$ of the take-home pay is $\frac{11}{15}$ of the take-home pay. Hence the amount spent on food and rent has to be:

2/5 + 1/3 = 11/15 requires that 2/5, 1/3, and 11/15 modify the same noun. In this problem each modifies "of the take-home pay"

$$\frac{11}{15} \text{ of } \$300 = (\$300 \div 15) \times 11$$
$$= \$20 \times 11$$
$$= \$220$$

In other words, a rate of \$11 out of each \$15 is the same as a rate of \$220 out of every \$300. As a check:

$$\frac{11}{15} = \frac{11 \times 20}{15 \times 20} = \frac{220}{300}$$

5.

We could make a chart similar to the one we used in the text and find all the prime numbers between 2 and 60. However, there is a quicker way. First of all, we are only considering the numbers:

50, 51, 52, 53, 54, 55, 56, 57, 58, 59, and 60.

Any number that ends in 0, 2, 4, 6, or 8 is divisible by 2 and hence isn't a prime number. That is we can eliminate 50, 52, 54, 56, 58, and 60 to leave us with:

51, 53, 55, 57, 59

A number is divisible by 3 if the sum of its digits is divisible by 3. 5 + 1 = 6 and 5 + 7 = 12. Hence 51 and 57 are divisible by 3 and can be deleted from our list, leaving us with:

We could divide each of the numbers by 3 and see which are multiples of 3 but this is a shortcut. As a check 51 = 3 X 17 and 57 = 3 X 19

53, 55, and 59

5. (cont)

Since any number that ends in 5 is divisible by 5, we can delete 55 from our list leaving us with:

53 and 59.

Since 8 X 8 is 64 and 64 is greater than either 53 or 59, we know that at least one of the factors of either 53 or 59 must be less than 8.

That is, the product of two numbers each of which is greater than 8 will be greater than 8 X 8 or 64.

We already know that neither 53 nor 59 is divisible by 2, 3, or 5 because if they had been they would have already been deleted from our list. 7 is the only prime number left to worry about because the next prime number, 11, is greater than 8. Since neither 53 nor 59 is a multiple of 7, we conclude that 53 and 59 are prime numbers.

We don't have to worry abou[t] 6 being a factor because 6 is 2 X 3 and we already know that 2 and 3 are not factors of 53 or 59.

7)53 = 7 R4   7)59 = 8 R3

In summary, since 8 X 8 = 64, any number less than 64 (other than 1) is either a prime number or else it has 2, 3, 5, or 7 as a factor.

6.

Since 23,100 ends in a 0 it is divisible by 2. In fact, 23,100 = 2 X 11,550. 11,550 ends in 0 so it too can be divided by 2. 11,550 = 2 X 5,775.

Since 5,775 ends in 5 it is divisible by 5. In fact 5,775 = 5 X 1,155. 1,155 ends in 5 so it too is divisible by 5. 1,155 = 5 X 231.

Numbers ending in 0 are als[o] divisible by 5. So we coul[d] have written that:
23,100 = 5 X 4,620
The important thing is that when we finish, the final prime factorization comes ou[t] to be the same, no matter what steps you use.

The sum of the digits in 231 is 2 + 3 + 1 or 6, which is divisible by 3. Hence 231 is divisible by 3. In fact, 231 = 3 X 77; and we hopefully recognize 77 as 7 X 11.

5 + 7 + 7 + 5 = 24 which is divisible by 3, Hence 5,775 is divisible by 3 but it is easier to recognize 5 as a factor than 3.

6. (cont)

"Stacking the factors" we have:

```
2)23,100
2)11,550
 5)5,775
 5)1,155
    3)231
     7)77
       11
```

Since 11 is a prime number, we have completed the factoring. We have:

23,100 = 2 X 2 X 5 X 5 X 3 X 7 X 11

and if we place the factors in increasing order we have:

23,100 = 2 X 2 X 3 X 5 X 5 X 7 X 11

Note

The prime factorization of 23,100 allows us to find all the factors--not just the prime factors--of 23,100. Namely, any combination of the factors in the prime factorization is also a factor. For example, we could choose both 2's and a 3 to get 2 X 2 X 3 or 12. Or we could take a 3, 5, and 7 to get 3 X 5 X 7 or 105. Whatever factors we pick, the other factor comes from what's left. For example if we pick 3, 5, and 7, we're left with 2, 2, 5, and 11. 2 X 2 X 5 X 11 = 220. 105 X 220 is 23,100

23,100 = 12 X 1,925

23,100 =105 X 220

In picking a prime factor of 23,100 we have two choices; either we pick it or we don't. Each of these 2 choices has to be made 7 times, one for each prime factor of 23,100. Hence there are 2^7 or 128 different factors of 23,100. We will not list them all here, but 12, 1,925, 105, and 220 are among them.

7.

(a) One way to begin is by obtaining the prime factorization of each number. Leaving the details to you, we have:

12 = 2 X 2 X 3

18 = 2 X 3 X 3

45 = 3 X 3 X 5

So be a multiple of 12 a number must have

7. (a) (cont)

the form:

2 X 2 X 3 X ________ (1)

Any whole number can go in the blank. The resulting product will then be divisible by 2 X 2 X 3.

To be a multiple of 18 a number must have the form:

2 X 3 X 3 X ________ (2)

So starting with (1) we need only annex another 3 to make sure that we have a multiple of 18 as well. In other words:

2 X 2 X 3 X 3 is the least common multiple of 12 and 18.

If one of the 2's were omitted the product wouldn't be divisible by 12 and if one of the 3's were omitted, the product wouldn't be divisible by 18.

Now we have to figure out what additional factors must be annexed to 2 X 2 X 3 X 3 to make sure that we have a multiple of 45. Well, to be a multiple of 45, the number must have the form:

3 X 3 X 5 X ________ (3)

The two factors of 3 in (3) are already contained in 2 X 2 X 3 X 3, so all we have to do is annex the 5, to get:

$$\begin{aligned} 2 \times 2 \times 3 \times 3 \times 5 &= 2^2 \times 3^2 \times 5 \\ &= 4 \times 9 \times 5 \\ &= 36 \times 5 \\ &= 180 \end{aligned}$$

In particular, 180 = 12 X 15 = 18 X 10 = 45 X 4. That is, 180 is the 15th multiple of 12, the 10th multiple of 18, and the 4th multiple of 45. *Any other common multiple of 12, 18, and 45 is also a multiple of 180.*

Do you see what's happening here? We could have found a common multiple of 12, 18, and 45 simply by finding the product 12 X 18 X 45 or 9,720. If we do this we're using too many factors. In fact we're using every prime factor in the sense that 12 X 18 X 45 means (2 X 2 X 3) X (2 X 3 X 3) X(3X3X5)

However we need only use each prime factor the greatest number of times it occurs in any one factorization. For example, a common multiple of 12, 18, and 45 doesn't need more than two factors of 3; but if it has less that 2 factors of 3, it will not be divisible by either 18 or 45, both of which have two factors of 3 in their prime factorization.

7. (b)

A major aim of this problem is to show that the associative property for addition holds for common fractions as well as for whole numbers. Let's use the result of part (a) to get common denominators.

$$\frac{5}{12} = \frac{5 \times 15}{12 \times 15} = \frac{75}{180}$$

$$\frac{7}{18} = \frac{7 \times 10}{18 \times 10} = \frac{70}{180}$$

$$\frac{2}{45} = \frac{2 \times 4}{45 \times 4} = \frac{8}{180}$$

Replacing each fraction by its equivalent, we get:

$$\left(\frac{5}{12} + \frac{7}{18}\right) + \frac{2}{45} = \left(\frac{75}{180} + \frac{70}{180}\right) + \frac{8}{180}$$

$$= \frac{(75 + 70)}{180} + \frac{8}{180}$$

$$= \frac{145}{180} + \frac{8}{180}$$

$$= \frac{145 + 8}{180}$$

$$= \frac{153}{180}$$

See why parts (b) and (c) will have the same answer? (75 + 70) + 8 = 75 + (70 + 8) as long as 75, 70, and 8 modify the same noun. In this exercise they each modify 180ths.

Notice that 1 + 5 + 3 = 9 and 1 + 8 + 0 = 9. Hence both 153 and 180 are divisible by 3. This means that our answer is not in lowest terms. In fact:

$$\frac{153}{180} = \frac{3 \times 3 \times 17}{3 \times 3 \times 20}$$

$$= \frac{\cancel{3} \times \cancel{3} \times 17}{\cancel{3} \times \cancel{3} \times 20}$$

$$= \frac{17}{20}$$

When we add two common fractions that are in lowest terms the sum might not be in lowest terms. For example, 5/12 and 1/12 are in lowest terms, but their sum is 6/12 which can be reduced to 1/2.

We didn't complete the prime factorization of 20 because it has no factor in common with 17.

As we shall show in part (c) we could have worked within the parentheses first and thus worked with only two fractions at a time.

7.(b) (cont)

But no matter how we proceed, the correct answer will always be $\frac{153}{180}$. To help visualize this from a practical point of view, suppose one partner in a firm owns $\frac{5}{12}$ of the firm. That is, he keeps 5 out of every 12 dollars the firm makes. At that rate, he would keep \$75 out of each \$180. If a second partner owns $\frac{7}{18}$ of the firm, he keeps \$7 out of each \$18, or \$70 out of each \$180. If the third partner owns $\frac{2}{45}$ of the company he keeps \$8 out of each \$180 the company earns. So between them the three partners keep (\$75 + \$70 + \$8)--or \$153--out of each \$180 the company earns. From another point of view, the rest of the ownership combined earns \$27 (that is, \$180 - \$153) out of each \$180 the company earns.

We're using 153/180 rather than 17/20 because it makes the arithmetic in the illustration easier to perform.

While 2/45 doesn't seem like a very big ratio, it could mean a lot of money if the company's earnings were big. For example, if the company earned \$45,000,000 this ratio would amount to \$2,000,000.

(c) The answer has to be $\frac{17}{20}$, but let's do it the "long way" to check the various steps.

18 = 2 X 3 X 3 and 45 = 3 X 3 X 5, so the least common multiple of 18 and 45 is 2 X 3 X 3 X 5 or 90. Hence:

$$\frac{7}{18} = \frac{7 \times 5}{18 \times 5} = \frac{35}{90}$$

$$\frac{2}{45} = \frac{2 \times 2}{45 \times 2} = \frac{4}{90}$$

Therefore:

$$\frac{7}{18} + \frac{2}{45} = \frac{35}{90} + \frac{4}{90} = \frac{39}{90}$$

Don't forget the "old-fashioned" way. List the multiples of 18 and 45 to get: 45, 90, 135, 180, 225,.
18, 36, 54, 72, 90,...
and see that 90 is the least common multiple.

7. (c) (cont)

Hence:

$$\frac{5}{12} + \left(\frac{7}{18} + \frac{2}{45}\right) =$$

$$\frac{5}{12} + \frac{39}{90}$$

We already have found that $90 = 2 X 3 X 3 X 5$. To find a multiple of 90 that is also a multiple of 12 we need another factor of 2 because $12 = 2 X 2 X 3$

Now:

$$\frac{5}{12} = \frac{5 X 15}{12 X 15} = \frac{75}{180}$$

and

$$\frac{39}{90} = \frac{39 X 2}{90 X 2} = \frac{78}{180}$$

Therefore:

$$\frac{5}{12} + \frac{39}{90} = \frac{75}{180} + \frac{78}{180}$$

$$= \frac{75 + 78}{180}$$

$$= \frac{153}{180} \quad (\text{or } \frac{17}{20})$$

8.

The aim of this exercise is to reaffirm the fact that the operation of subtracting common fractions is not associative. Namely if you compare parts (a) and (b) you'll notice that both parts look identical if the grouping symbols are omitted. That is, without the parentheses, both (a) anf (b) look like:

$$\frac{4}{7} - \frac{1}{3} - \frac{2}{11}$$

(a) In part (a) we'll first use the fact that 21 is the least common multiple of 7 and 3. In fact:

If m and n are prime numbers, m X n is always the least common multiple of m and n.

$$\frac{4}{7} = \frac{4 X 3}{7 X 3} = \frac{12}{21}$$

$$\frac{1}{3} = \frac{1 X 7}{3 X 7} = \frac{7}{21}$$

So $\frac{4}{7} - \frac{1}{3} = \frac{12}{21} - \frac{7}{21} = \frac{5}{21}$

8. (a) (cont)

So $(\frac{4}{7} - \frac{1}{3}) - \frac{2}{11} = \frac{5}{21} - \frac{2}{11}$

Now $\frac{5}{21} = \frac{5 \times 11}{21 \times 11} = \frac{55}{231}$

and $\frac{2}{11} = \frac{2 \times 21}{11 \times 21} = \frac{42}{231}$

Putting it all together, we have:

$$(\frac{4}{7} - \frac{1}{3}) - \frac{2}{11} = \frac{5}{21} - \frac{2}{11}$$
$$= \frac{55}{231} - \frac{42}{231}$$
$$= \frac{13}{231}$$

Even though 21 is not a prime number, it has no common factor (except 1) with 11. In this case we say that 21 and 11 are relatively prime. *If m and n are relatively prime, m X n is the least common multiple of m and n.*

(b) In this case, we'll first use the fact that 33 is the least common multiple of 3 and 11. Namely:

$$\frac{1}{3} = \frac{1 \times 11}{3 \times 11} = \frac{11}{33}$$
$$\frac{2}{11} = \frac{2 \times 3}{11 \times 3} = \frac{6}{33}$$

Therefore:

$$\frac{4}{7} - (\frac{1}{3} - \frac{2}{11}) = \frac{4}{7} - (\frac{11}{33} - \frac{6}{33})$$
$$= \frac{4}{7} - \frac{5}{33}$$

Since 7 and 33 are relatively prime (that is, the prime factorization of 33 is 3 X 11 which doesn't contain 7 as a prime factor) the least common multiple is 7 X 33 or 231:

$$\frac{4}{7} = \frac{4 \times 33}{7 \times 33} = \frac{132}{231}$$
$$\frac{5}{33} = \frac{5 \times 7}{33 \times 7} = \frac{35}{231}$$

It isn't nearly as important to find the least common multiple as it is to find a common multiple. All the least common multiple does is help to simplify the arithmetic. But with any common multiple, we'd still get the same answer eventually.

Hence:

$$\frac{4}{7} - (\frac{1}{3} - \frac{2}{11}) = \frac{4}{7} - \frac{5}{33}$$
$$= \frac{132}{231} - \frac{35}{231}$$

8. (b) (cont)

$$= \frac{132 - 35}{231}$$

$$= \frac{97}{231}$$

Comparing the answers for parts (a) and (b), we see that when we subtract three of more common fractions, the grouping makes a crucial difference.

9.

At first glance this may seem like a harder problem than it really is. Namely, it isn't too simple to see how 667 factors. *But the important point is that it doesn't matter!* What is important is that the numerator (46) factors as 2 X 23 where both 2 and 23 are prime numbers.

The key is that we can only cancel <u>the same</u> *factor from both numerator and denominator. Since 2 and 23 are the only factors of the numerator, it makes no difference what other factors the denominator has.*

Now we know that 667 isn't divisible by 2 because it ends in 7. Is 667 divisible by 23? All we have to do is check, as shown in the margin. We find indeed that 23 X 29 = 667. Therefore:

$$\begin{array}{r} 29 \\ 23\overline{)667} \\ -\ \underline{46} \\ 207 \\ -\ \underline{207} \\ 0 \end{array} \qquad \begin{array}{r} 29 \\ \times\ \underline{23} \\ 87 \\ +\ \underline{58} \\ 667 \end{array}$$

$$\frac{46}{667} = \frac{23 \times 2}{23 \times 29}$$

$$= \frac{\cancel{23} \times 2}{\cancel{23} \times 29}$$

$$= \frac{2}{29}$$

See the point here? We don't have to find the prime factorization of 667 to reduce 46/667 to lowest terms All we have to do is check whether 2 or 23 are factors of 667.

9. (cont)

We chose 667 as a denominator simply to have a number large enough so that it was not obvious what its prime factorization was; but the size of the denominator is not important. For example, look at:

$$\frac{2}{1,234,987,001} \qquad (1)$$

Don't waste time looking for the prime factorization of 1,234,987,001 to see whether (1) can be reduced. Rather observe that the numerator is 2, which is a prime number. Hence the only candidate for a common factor of both the numerator and denominator of (1) is 2. But the fact that 1,234,987,001 ends in a 1 means that the denominator is not divisible by 2. Threrefore we know that (1) is in lowest terms.

In summary, then; whenever we want to reduce a cumbersome common fraction to lowest terms, it is sufficient to be able to factor either the numerator or the denominator. Once we factor one of the two, we know all the candidates for <u>common</u> factors of both the numerator and the denominator.

10.

Perhaps the key point of this problem is to make sure that you realize that there are many different ways of saying to take $\frac{2}{3}$ of a number. The advertisement might scare off the average buyer if it said "Pay for only $\frac{2}{3}$ of what you buy." Yet that's what it says! Namely if you buy 3 but only pay for 2, then at this rate if you buy 6 you'll only pay for 4; if you buy 9, you'll only pay for 6; and so on. This is exactly what it means to say that you'll pay for $\frac{2}{3}$ of what you buy.

Yet some ads say this in a somewhat different way. For example, you may have seen ads that read "1/3 off the list price". "1/3 off" means that for each \$3, \$1 is deducted. If \$1 is deducted you still are paying \$2 for each \$3 of the list price. Hence the ad could have read: "Buy \$3 worth, but pay for \$2 worth"

Since we are buying 750 items, the number that we have to pay for is:

$$\frac{2}{3} \text{ of } 750$$

or

$$2 \times (750 \div 3) =$$

$$2 \times 250 =$$

$$500$$

Each item costs 39¢, so we have to pay 39¢, 500 times or 39¢ X 500.

$$39¢ \times 500 = 19{,}500¢$$

$$= \$195$$

Note that there are other ways of doing this problem.

Notice the use of rates here. There are 100 cents per dollar. So we divide the number of cents by 100 to get the number of dollars.

$$\begin{array}{r} \$\,195 \\ 100\overline{)19500}\,¢ \\ \underline{-100} \\ 950 \\ \underline{-\ 900} \\ 500 \\ \underline{-\ 500} \end{array}$$

10. (concluded)

For example, notice that paying for $\frac{2}{3}$ means that you are also paying for only $\frac{2}{3}$ of the cost of each item. Since $\frac{2}{3}$ of 39 is 26, you are paying a true price of 26¢ per item. Altogether you are buying 750 items, so you pay 26c, 750 times or 26c X 750 or $175.

2 X (39 ÷ 3) = 2 X 13 = 26

Don't use 500 here. We've reduced the price by this method--not the number of items you pay for.

A Note on Marketing

Which sounds more impressive: "Buy 3 but pay for only 2" or "Regularly 39¢ each. Sale price 26¢ each"? Mathematics only tells us that the two offers mean the same thing. Marketing tells us how to "package" the information to make it more desirable to the consumer.

My own guess is that :"Buy 3, Pay for 2" is a better ad, but that the consumer should keep in mind that the sale price is 26¢ each.

This concludes our discussion of the Self-Test. Hopefully the selection of problems has allowed you to get a better feeling about common fractions and to see how the arithmetic of common fractions is a direct outgrowth of the arithmetic for whole numbers.

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Step 7:

Do Self-Test 4, Form B on the next page.

Self-Test 4, Form B

ANSWERS:

1. Which of the following is not a common fraction:

$$\frac{5}{5}, \frac{3}{0}, \frac{2}{5}, \frac{9}{4}, \text{ or } \frac{7}{0}?$$

1. ________

2. Which of the following common fractions is equivalent to 3/4:

$$\frac{3+3+3}{4+4+4}, \frac{3+5}{4+5}, \frac{4}{3}, \frac{3 \times 4}{4 \times 3}, \text{ or } \frac{3 \times 3}{4 \times 4}?$$

2. ________

3. List the following common fractions in the order from least to greatest.

$$\frac{2}{3}, \frac{3}{4}, \frac{5}{12}, \frac{7}{15}, \text{ and } \frac{17}{24}$$

3. ________

4. (a) How much is $\frac{3}{7} + \frac{1}{5}$? Make sure the sum is in lowest terms.

   (b) You spend 3/7 of your weekly take-home pay on rent and 1/5 on food. If your weekly take home pay is $210, how much do you spend each week on rent and food?

4. (a) ________

   (b) ________

5. List all the prime numbers between 60 and 70.

5. ________

6. Write 31,680 as a product of prime numbers.

6. ________

7. (a) Find the least common multiple of 15, 35, and 63.

   (b) Express (4/15 + 2/35) + 8/63 as a common fraction in lowest terms.

   (c) Express 4/15 + (2/35 + 8/63) as a common fraction in lowest terms.

7. (a) ________

   (b) ________

   (c) ________

8. (a) Express (5/6 - 3/7) - 1/5 as a common fraction in lowest terms.

   (b) Express 5/6 - (3/7 - 1/5) as a common fraction in lowest terms.

8. (a) ________

   (b) ________

9. Reduce $\frac{106}{2,173}$ to lowest terms.

9. ________

10. You want to buy 175 items that usually cost 35¢ each. A special sale advertises: "Buy 7; pay for only 5!" How much will the 175 items cost you during the special sale?

10. ________

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 4, Form B

1. $\frac{7}{0}$

2. $\frac{3 + 3 + 3}{4 + 4 + 4}$

3. $\frac{5}{12}$, $\frac{7}{15}$, $\frac{2}{3}$, $\frac{17}{24}$, and $\frac{3}{4}$

4. (a) $\frac{22}{35}$ (b) \$132

5. 61 and 67

6. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 11$ or $2^6 \times 3^2 \times 5 \times 11$

7. (a) 315 (b) $\frac{142}{315}$ (c) $\frac{142}{315}$

8. (a) $\frac{43}{210}$ (b) $\frac{127}{210}$

9. $\frac{2}{41}$

10. \$43.75 (4,375 cents)

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If you did each problem in Form B correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8.

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Step 8:

View the solutions for Self-Test 4, Form B on Videotape Lecture 4S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. *Feel free to rewind the tape at any time to restudy any problems that gave you difficulty.*

Step 9:

Do Self-Test 4, Form C on the next page.

Self-Test 4, Form C

ANSWERS:

1. Which of the following is not a common fraction: $\frac{9}{4}$, $\frac{4}{9}$, $\frac{0}{3}$, $\frac{1}{0}$, or $\frac{6}{6}$?

1. ____________

2. Which of the following common fractions is equivalent to 4/7: $\frac{7}{4}$, $\frac{4 \times 4}{7 \times 7}$, $\frac{4 \times 2}{7 \times 2}$, $\frac{4 + 3}{7 + 3}$, or $\frac{4 \times 7}{7 \times 4}$?

2. ____________

3. List the following common fractions in the order of least to greatest. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{12}$, $\frac{11}{30}$, and $\frac{13}{40}$.

3. ____________

4. (a) How much is $\frac{2}{5} + \frac{1}{6}$?

   (b) You spend 2/5 of your weekly take-home pay on rent and 1/6 on food. If your weekly take-home pay is $150, how much do you spend on food and rent each week?

4. (a) ____________

   (b) ____________

5. List all the prime numbers between 70 and 80.

5. ____________

6. Write 11,700 as a product of prime numbers.

6. ____________

7. (a) Find the least common multiple of 14, 30, and 35.

   (b) Express (5/14 + 7/30) + 4/35 as a common fraction in lowest terms.

   (c) Express 5/14 + (7/30 + 4/35) as a common fraction in lowest terms.

7. (a) ____________

   (b) ____________

   (c) ____________

8. (a) Express (3/4 - 2/5) - 1/7 as a common fraction in lowest terms.

   (b) Express 3/4 - (2/5 - 1/7) as a common fraction in lowest terms.

8. (a) ____________

   (b) ____________

9. Reduce $\frac{166}{6,557}$ to lowest terms.

9. ____________

10. You want to buy 600 items that usually cost 20¢ each. A special sale advertises: "Buy 5; pay for only 4!" How much will the 600 items cost you during the special sale?

10. ____________

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 4, Form

1. $\frac{1}{6}$

2. $\frac{4 \times 2}{7 \times 2}$

3. $\frac{13}{40}$, $\frac{11}{30}$, $\frac{5}{12}$, $\frac{4}{9}$, and $\frac{2}{3}$

4. (a) $\frac{17}{30}$ (b) $85

5. 71, 73, and 79

6. $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 13$ or $2^2 \times 3^2 \times 5^2 \times 13$

7. (a) 210 (b) $\frac{74}{105}$ (c) $\frac{74}{105}$ ($\frac{148}{210}$ reduces to $\frac{74}{105}$

8. (a) $\frac{29}{140}$ (b) $\frac{69}{140}$

9. $\frac{2}{79}$

10. $96 (9,600 cents)

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THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #4.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #5.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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